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ZPA and Other Mysteries of Seismic Testing

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EARTHQUAKE SHAKE TABLES

Seismic testing, aka earthquake testing, of structural models, mechanical equipment and electrical components is most often performed using shake tables to verifying the ability of these items to survive earthquakes. This verification is essential for components used in critical structures such as hospitals, police and fire stations, telecommunication centers, and nuclear and conventional power plants. Seismic testing can be performed on shake tables with 1, 2, or 3 degrees-of-freedom. Increasingly, regulators are asking for testing on 3-degree-of-freedom tables (3 DOF) which are also known as triaxial tables, as they most realistically reproduce earthquake motions. Figures 1 and 2 present photographs of two typical triaxial seismic shake tables. One is driven by electric actuators, the other by servo hydraulic actuators.

Figure 1: Typical Electric Drive Triaxial Shake Table (delivered by ANCO Engineers to Westinghouse, New Stanton, Pennsylvania)

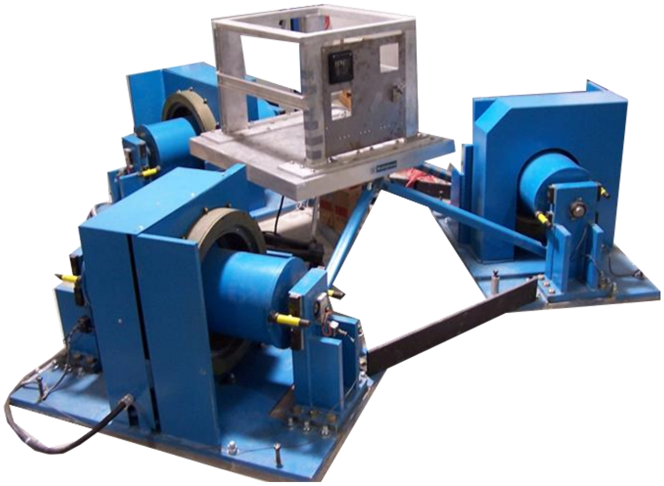


Figure 2: Typical Servo Hydraulic Triaxial Shake Table (Delivered by ANCO Engineers to AZZ/NLI, Fort Worth Texas)

EARTHQUAKE RESPONSE SPECTRA

During triaxial table testing, there is motion simultaneous in all 3 DOF (x-horizontal, y-perpendicular horizontal, and z-vertical) normally lasting about 30 seconds. The motion in each axis is typically required to be different ("independent") from the motion in the other axes. While these motions must be actual time histories, the basis for defining what these time histories are uses a concept called "Response Spectra" to describe the motions in a frequency domain. Response Spectra are extremely useful in defining the nature of the earthquake time histories but involve a number of concepts and definitions that mystify the lay person and occasionally confuse even the experts. We once heard a test engineer mumble:

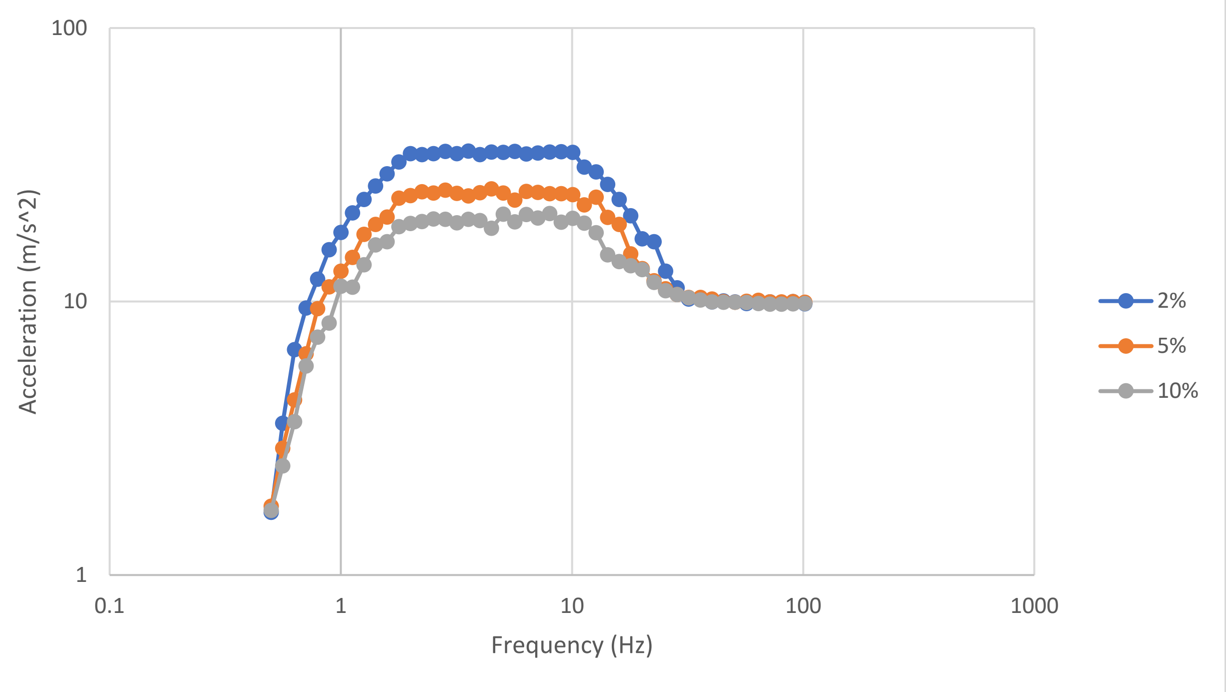
- "Why does the ZPA not match the peak acceleration in the time history?"
- "What the heck does that mean and does this make any sense?"

To explain this, we should start with the definition of Response Spectra or RS (which are known in some circles as a Shock Spectra). Also note that the singular is called a Response Spectrum and the plural are called Response Spectra. An RS is defined as the peak response (typically acceleration) experienced by a single degree-of-freedom oscillator when exposed to a dynamic base motion such as an earthquake. In earthquake testing, one is typically not interested if the maximum motion is in the positive or negative direction. What is important is the maximum absolute value. (In some studies, it may be of interest what happens in the negative versus positive direction and one can speak about the Positive RS and Negative RS, but this will not be of concern for the rest of this article).

The value of the RS will depend on the resonant frequency of the oscillator (f in Hz.), the fraction of critical damping of the oscillator (β), and the nature of the dynamic motion. The damping characteristic of a structure is expressed as a fraction or percent of critical damping. If an oscillator has critical damping (β=100%) or more than critical damping (β>100%), it will not oscillate, but will exponentially decay in response to a step motion. If the damping is less than critical (β<100%), it will oscillate as it responds to a step motion. Most mechanical and civil structures have damping values far less than critical, typically in the range of β=0.005-0.3 (0.5-30%. Typically, the RS is plotted as a graph with frequency (f) on the horizontal axis and the acceleration, commonly known as Spectral Acceleration (SA), on the vertical axis. It is important to note that SA is not the acceleration of the base motion (earthquake) but rather the peak absolute response of an oscillator with natural frequency f and damping β when exposed to the given earthquake base motion.

Figure 3 illustrates the RS for a typical earthquake. Note that, at low resonant frequencies, below about 2 Hz, the RS is relatively small. Why is this? Imagine a very flexible oscillator, meaning that it has a resonant frequency below about 2 Hz. During an earthquake, this flexible oscillator acts like an isolation system and much of the ground acceleration is not transmitted into the oscillator. Therefore, the RS falls off below about 2 Hz. Granted, the relative displacement to the ground is large, but the spectral acceleration of the oscillator is small. Note that, in Figure 3, there are several curves for different values of damping. This is because SA increases as damping decreases.

Figure 3: Typical RS with damping values of 2%, 5%, and 10%, most energy from 2-10 Hz and asymptote at 30 Hz.



Typically, between about 2 Hz and 10 Hz, the earthquake has a lot of energy and acceleration, so the RS is large in this frequency range. The energy in typical earthquakes tends to be smaller above 10 Hz and usually there is little energy above 30 Hz (but there can be exceptions). Therefore, the RS typically falls off for natural frequencies above about 10 Hz.

An interesting phenomenon occurs as the RS falls off as the resonant frequency reaches about 30 Hz. All the damping curves merge together, and they reach an asymptote value that does not increase as the resonant frequency increases to higher values. To understand the reason for this, consider an oscillator with a very high natural frequency (imagine a 1-inch cube of steel welded to the vibrating ground). As there is little earthquake energy above 30 Hz, the steel cube does not resonate and will just follow the ground acceleration. Therefore, the SA of a high frequency oscillator will simply be equal to the peak acceleration of the ground motion and this will be the same regardless of resonant frequency or damping. Therefore, the asymptote SA value of the RS will always be equal to the peak acceleration of the ground motion.

Now we must defer to our colleagues the Seismologist. For reasons best known to them, they prefer to talk about the natural Period of the oscillator (P) rather than its natural frequency (f). This is probably because seismologists are very concerned with the Period of the various seismic waves. Note that there is a simple relationship, of course, between P and f, that is P = 1/f. So, if an engineer is concerned with the high frequency asymptote, he might call it the “HFA”. But the seismologist dealt with these issues before the engineers. As frequency gets very large, Period gets very small and approaches zero. So, the seismologist likes to talk about the Zero Period Asymptote or “ZPA”. So, we have finally defined the ZPA. One last confusion is that many people think that the ZPA means “Zero Period Acceleration” instead of “Zero Period Asymptote”. Both interpretations are acceptable and in common use.

Note that, in Figure 3, the ZPA is equal about 1 g (9.81 m/s^2). Based on the above, it should be clear that the ZPA is equal to the peak acceleration of the underlying earthquake time history. There are no exceptions to this, so why do some people think this is not true?

WHY DO SOME PEOPLE THINK THAT THE ZPA CAN BE LARGER THAN THE PEAK EARTHQUAKE ACCELERATION?

Typically, the response spectra on a shake table are computed from the time histories measured by accelerometers on the shake table. A digital software program solves the equations of motion to compute the RS for many values of f and acceleration. This calculation is usually carried out and then plotted up to f=100 Hz. But sometimes it is only plotted up to f=50 Hz. Often this is sufficient to observe and define the ZPA but sometimes this is deficient and misinterpreted for either of two reasons:

1. The most common error occurs if the earthquake happens to have significant energy at frequencies higher than the plotting limit. So, let's suppose a particular earthquake is plotted up to 50 Hz but the time history happens to have some significant energy at 70 Hz. The response spectra may then look as shown in Figure 4. As can be seen, the plot tends to somewhat level off as one approaches 50 Hz. Some may therefore interpret that the ZPA is the value at 50 Hz (1.2 g). However, if one more wisely plotted the ZPA to 100 Hz, one will see that the true asymptote is not reached until about 80 Hz and has a value of about 1.0 g. Therefore, the true value of the ZPA is 1.0 g instead of 1.2 g as erroneously interpreted in Figure 4 if one only plotted up to 50 Hz. To avoid such misinterpretation, always plot the RS to a sufficiently high frequency to identify the true ZPA. Usually 100 Hz is sufficient (but see below).

2. The time histories used to drive a shake table are most often computed using a Spectrum Compatible Time History program. This is a digital program that uses a variety of different algorithms to create a time history that has an RS that closely but not exactly matches the RS required by the test program. This is usually called the Required Response Spectrum or RRS and more will be discussed on this later on. These programs are quite versatile and try very hard and generally do a good job to match the RRS. However, if the program user does not carefully limit the energy band that the program can use to match the RRS, the program can add a significant amount of high frequency energy. In such a case, the RS can appear to reach an asymptote below 100 Hz but, in fact, the actual asymptote might not occur, for example, until 200 Hz. Such a case is illustrated in Figure 5. To avoid such errors, make sure to limit the energy content of your Spectrum Compatible program to a reasonable value, such as 50 Hz, unless otherwise required (there are some hard-rock earthquakes, steam valve release vibrations, and aircraft impact vibrations, with energies in the 50 to 100 Hz range). Similarly, you can use analog or digital filters on your recorded shake table motions to make sure that there is no significant energy in your signal well below the maximum frequency of your RS plot.

Figure 4: Earthquake RS showing ZPA effect at 50 Hz vs 100 Hz. Fake ZPA at 50 Hz is about 1.2 g. Peak around 70 Hz. True ZPA is about 1.0 g.

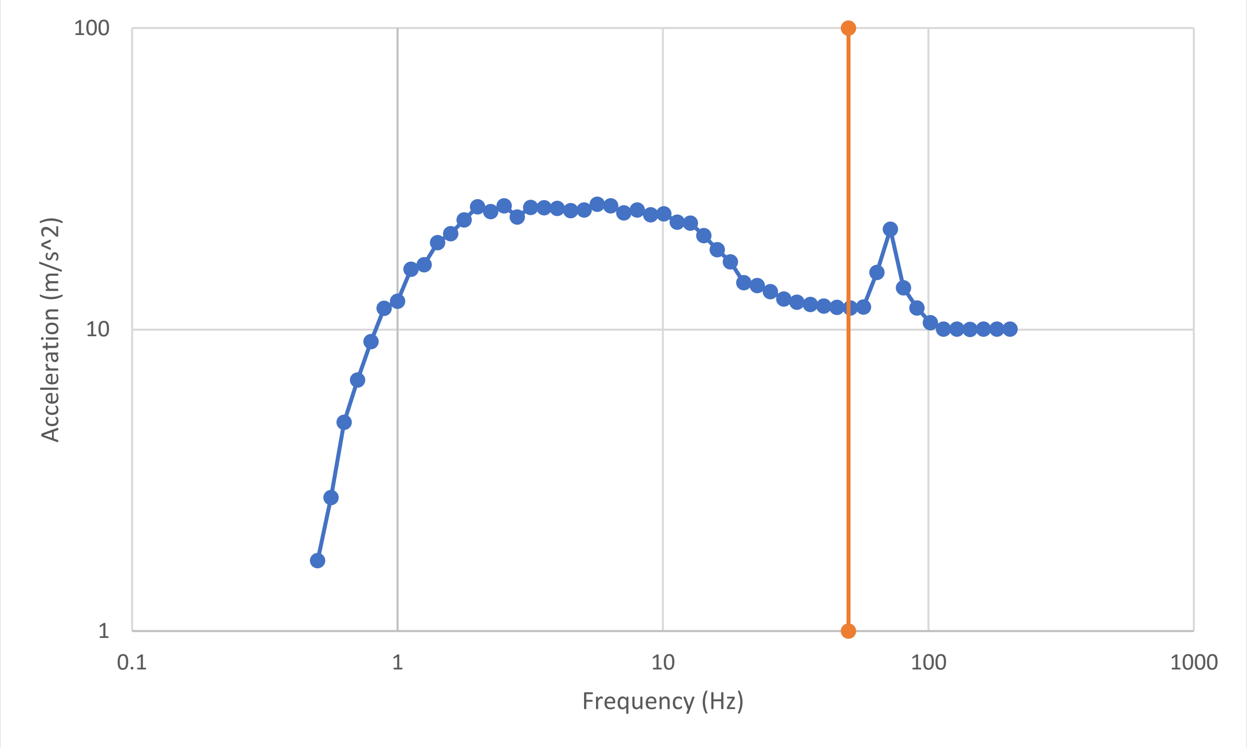
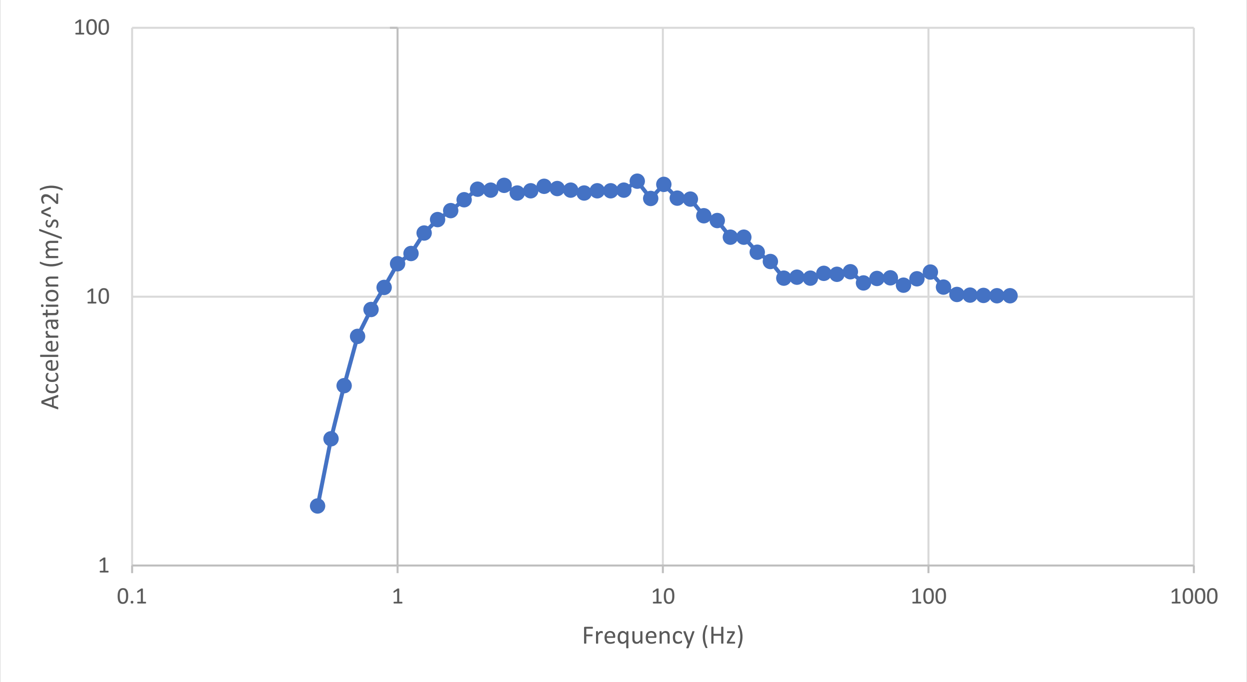


Figure 5: Example of false ZPA due to excessive high frequency energy. Real ZPA is about 1 g and appears to be about 1.2 g below 100 Hz.



Making one of the above two errors cause one to think that the ZPA is actually larger than the peak acceleration in your time history. But, mathematically, this cannot be the case and, if observed, it is likely that one of the above two errors has been made.

Hopefully the above discussion has clarified the meaning of a Response Spectra, the definition of the ZPA, why the ZPA must equal the peak acceleration of the earthquake time history, and how to avoid false ZPA readings.

RESPONSE SPECTRA BROADENING

An earthquake time history motion far away from any structure is called a Free Field Earthquake. This earthquake will have an RS. However, engineers testing equipment in a building structure are more interested in how the structure will modify the Free Field Earthquake to affect equipment in the structure. So, a structural engineer will create a dynamic model of the structure and use it to predict the time history motion at an elevated location in the structure. This motion, and the corresponding RS, will have more energy and amplitude than the free field earthquake, especially around the resonant frequencies of the structure and typically a larger ZPA. The computed RS of this motion is called a Floor Response Spectra or “FRS”. One may then consider reproducing this FRS on a shake table to test equipment that may be located on that particular floor of the building structure. There is one concern with this approach. Mainly, the engineer will recognize that the structural model may be somewhat inaccurate. For example, the engineer may predict that the first resonant frequency of the building is at 3 Hz whereas, in fact, it may be at 3.2 Hz. This means that the computed FRS will have a peak near 3 Hz whereas, in fact, the real FRS will have a peak near 3.2 Hz. To address this issue, the structural engineer will shift the peak of the FRS ±10% to lower and higher frequencies to “broaden” the response spectrum to account for model uncertainty.

RESPONSE SPECTRA ENVELOPING

The engineer may consider that the equipment might be placed on different floors of the structure (or even on different floors in a different structure) and so the engineer may plot several broadened FRS on the same graphs and then sketch an approximate envelope of all the FRS with relatively few line segments. The engineer will then define this broadened and enveloped FRS by specifying the frequency and amplitude of the end points of these line segments also known as “Break Points”. The resulting simplified FRS typically represents a much more energetic and damaging earthquake than any of the individual originating FRS. The shake table test engineer then has the task of reproducing this more energetic and hypothetical combined FRS. This is not always an easy task. This is also the cause when developing in-structure RS since components to be tested can be found in different locations within the equipment.

PHYSICALLY AND MATHEMATICALLY IMPOSSIBLE RESPONSE SPECTRA

In addition to the broadening and enveloping of FRS discussed above, these FRS may also be further modified by various organizations and regulatory groups. While these modifications can be useful, and are well intended, they sometimes lead to a hypothetical FRS that cannot be physically or mathematically realized with any time history or shake table. We will cite two cases here. The first case deals with the maximum possible ratio between the peak spectral acceleration and the ZPA. The mathematic maximum possible ratio would come from an oscillator being excited for long duration at its resonant frequency by a single frequency sinusoid. In this case, the maximum ratio is $1/(2\beta)$. This ratio is also known as Q. For example, if β is 5%, this ratio is 10 (Q=10). And so, the maximum possible spectral acceleration on a 5% RS is 10x the ZPA and often less because a finite duration earthquake cannot be a pure sine wave. Due to the broadening, enveloping, and regulatory manipulations of the FRS, sometimes this rule is violated. In these cases, the shake table engineers must sometimes artificially increase the ZPA in order to match the peak spectral acceleration. Similarly, sometimes the spectral acceleration ends up too close to the ZPA and the engineer must over-test the peak spectral acceleration in order to be able to use a realizable time history. These problems are often increased because, by the time the shake table engineer receives an FRS plot, it may have been faxed, copied, and scanned a dozen times.

STATIONARITY AND INDEPENDENCE

In the 1970s, seismic test engineers and regulators became concerned that multi-axis motion on a shake table may not have similar properties to multi-axis motion on a structure during an actual earthquake. There was some concern that this might result in an under-test of the test item. For example, what if the shake table motion had all the low frequency at the beginning of the earthquake and the high frequencies at the end? What if the motions on the shake table looked too much like a sinusoid and did not contain multiple frequencies? What if the motions in two perpendicular directions on the shake table were too similar thereby creating a vector type motion instead of a random type motion? These phenomena were thought to be “non-earthquake-like”. These issues were investigated by several researchers (Daniel D. Kana of Southwest Research Institute (SWRI), in particular) and they came up with various tests and criteria to check if the shake table motion was suitably “earthquake-like”. They did this by studying certain statistical and spectral properties of multi-axis real measured earthquakes. They reasoned that earthquake motions on the shake table should have similar “Independence” and “Stationarity” as can be seen in real earthquakes.

INDEPENDENCE OF TWO TIME HISTORIES USING CORRELATION, COEFFICIENT, AND COHERENCE

The independence of two perpendicular directions of motion can be defined either by their Cross-Correlation or by their Coherence. The Cross-Correlation (also called correlation coefficient) is a single number ranging between minus 1 and plus 1. If the Cross-Correlation is 0, it means that the two-time histories are near totally independent (they are very different from each other). If the Cross-Correlation approaches plus or minus 1, then it means the two-time histories are nearly identical and dependent, although they may differ by a single constant multiplier. From the studies on typical earthquakes, a Cross-Correlation with absolute value of 0.3 or less is typical. So, one criterion placed on pairs of perpendicular table motions is that their absolute value of their Cross-Correlation be 0.3 or less. Note that, in computing the Cross-Correlation, it is necessary to do so as one shifts the two-time histories from each other and determine what shift in time creates the largest absolute value of the Cross-Correlation. This maximum value determined by time shifting is the value that must be 0.3 or less for time histories to be considered sufficiently independent. The time shift is often referred to as delta Tau and is a value in seconds. In almost all instances the absolute value of delta Tau is <1s.

The independence of two-time histories can also be determined by computing the Coherence between the two-time histories. The Coherence is a function of frequency and has an amplitude between 0 and 1. If the Coherence is near 1, it means the two-time histories are very similar at that particular frequency. If the Coherence is near 0, it means that the two-time histories are not very similar at that particular frequency. The studies on real earthquakes indicated that the Coherence between perpendicular time histories is typically 0.5 or less. Therefore, another way to specify the required degree of independence of shake table motions is to require that the Coherence be equal to or less than 0.5.

Many current earthquake testing regulations (such as IEEE STD 344) require that shake table motions be sufficiently independent such that either the Cross-Correlation be equal to or less than 0.3 or that their Coherence be equal to or less than 0.5 in the frequency range of interest to the test. Only one criterion must be met, not both. But it’s acceptable if both are met.

TIME HISTORY STATIONARITY

The Stationarity of a time history relates to how its frequency and amplitude content changes over time. The studies on real earthquakes, by Kana and others, involve breaking the earthquake up into several shorter portions (such as 6 sections, 5 seconds long, for an earthquake with a total duration of 30 seconds) and evaluating the Power Spectral Density (PSD) of each segment. The PSD gives a measure of the energy distribution and amplitude over the earthquake frequency range of interest. One can then compare the PSD of each segment with the average PSD of the entire earthquake. Kana reasoned that, if the PSD of the individual segments did not vary much from the average PSD, then the time history can be considered sufficiently stationary. From studies of actual earthquakes, Kana determined typical amounts by which the individual PSD exceeded the average PSD (RMAX) or was smaller than the average PSD (RMIN). Current regulations assure that the time history is sufficiently stationary by requiring that the RMAX and RMIN criteria be met. In IEEE-344, RMAX = 2.8 and RMIN=0.17.

NOT QUITE DONE YET

As a final thought, note that certain situations can give the shake table engineer difficulties in meeting the above criteria.

- **Excessive Coherence or Correlation due to building resonances affecting multiple directions.** As indicated above, some regulations require perpendicular time histories on a shake table to have coherence less than or equal to 0.3 or correlation less than or equal to 0.5 in order to be "earthquake like". However, when earthquake motions are amplified by building resonances that can affect motion in more than one perpendicular direction, motions can result in 2 perpendicular directions with a lot of energy near the same resonant frequencies. This can cause their correlation and coherence to be higher than the above criteria. This is totally normal and correct, and it may be very difficult to adjust the time histories to meet the above coherence/correlation criteria. The test engineer may have to point this out in certain extreme cases and ask for relaxation of the criteria. This can also be caused by RS enveloping.

- **Excessive correlation due to small RRS in the vertical direction.** Sometimes, the RRS called for have relatively large and similar amplitude for the horizontal directions (X and Y), but significantly smaller RRS in the vertical (Z) direction. In this case, it may be difficult to get the Z motion to have low coherence and correlation with the X and Y motions. This is because a small amount of leakage of the X and Y shake table motion leaking into the Z motion raises the coherence and correlation. It is often possible to decrease this correlation/coherence by multiplying the small vertical motion by a factor of 2 or more thereby diluting the effect of the leaked X and Y motions. The Z RRS still remains relatively small and this slight over-test in the vertical direction is a small price to pay in order to meet the coherence/correlation criteria.

- **Failure to exceed an RRS at a few frequencies.** Occasionally, the achieved Test Response Spectrum (TRS) may not envelope the RRS at every frequency point despite the best efforts of the test engineer. Some regulations do allow accepting 1 or 2 points where the TRS falls below the RRS by a small amount such as 10%. Check your regulations to see exactly what is allowed.

- **Failure of the TRS to Envelope the RRS at Low Frequencies.** This can be due to velocity or displacement limits of the table and usually occur below about 2-3 Hz. Some regulations can accept this as long as the equipment being tested can reasonably be expected to have no resonant frequencies below about 4-5 Hz. Check your regulations to see if this exception is allowed. Note that, if your test item has a resonance around 4 Hz and you tilted it down 90 degrees on its side, cantilevered off its base, some part of the test item will sag approximately 0.5 inch due to earth's gravity. This quick check can often be used to see if the above exception applies. If the sag is less than about 0.5 inch, then the resonant frequency of the test item will be above 4 Hz. (the derivation of this relationship is left to the reader!).

The authors, Paul Ibanez, Boaz Norton, and Michael Schulze have collectively designed and used over 200 seismic shake tables and Dr. Ibanez has been a member of the IEEE-344 committee for several decades. The authors would be pleased to discuss your seismic shake table questions and experiences with you.

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Figure 6: Typical Electric Drive Triaxial Shake Table (delivered by ANCO Engineers to Duke Energy, Charlotte, North Carolina)